Advanced Mechanics

Olaf Scholten
Kernfysisch Versneller Instituut
NL-9747 AA Groningen

Mid term exam, June 2009.
2 problems (total of 20 points).
The solution of every problem on a separate piece of paper with name and student number.

Problem 1 (7 pnts in total) (last years exam)
A damped oscillator $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = F(t)/m$ with $\omega_0^2 = 2\beta^2$ is driven by a force that is exponential in time,

$$F(t) = F_0 e^{-t/\tau} \text{ for } t > 0$$

with $\beta = 1/\tau$ and $F(t) = 0$ for $t < 0$.

4 pnts a. Show that the Greens function

$$G(t,t') = \frac{1}{m\omega_1} e^{-\beta(t-t')} \sin\omega_1(t-t'), \quad t \geq t'; \quad \text{and } G(t,t') = 0, \quad t \leq t'$$

is a solutions of $\frac{d^2}{dt^2}G(t,t') + 2\beta\frac{d}{dt}G(t,t') + \omega_0^2 G(t,t') = \delta(t-t')/m$ for $t \neq t'$ and $\omega_1^2 = \omega_0^2 - \beta^2$

3 pnts b. Give the expression for $x(t)$ using the Greens function.

3 pnts c. Solve for $x(t)$.

Problem 2 (10 pnts in total)
Find the shortest path between two points on a cylindrical surface, given by $x^2 + y^2 = r^2$.

2 pnts a. Give the form for the expression to minimize.

2 pnts b. Give the resulting Euler equations.

3 pnts c. Show that the general solution can be written as $z = a\phi + \phi_0$.

1 pnts d. Give a and ϕ_0 for the path from $p_1 = (x,y,z) = (r,0,0)$ to $p_2 = (0,r,1)$.

2 pnts e. Calculate the path length between these two points.